

# Black Hole Hawking Radiation

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## Abstract

Hawking radiation is black-body radiation that is predicted to be released by black holes – which make a black hole to glow like a piece of hot metal, due to quantum effects near the event horizon. It is named after the English theoretical cosmologist Stephen Hawking, who provided a theoretical argument for its existence in 1974. Hawking radiation – which is widely regarded as one of the first real steps toward a quantum theory of gravity and allows physicists to define the entropy of a black hole – reduces the mass and energy of black holes and is therefore also known as black hole evaporation. Because of this, black holes are not quite black! Instead, they glow slightly with photons, neutrinos, and other massive particles. They shrink and ultimately vanish. Micro black holes are predicted to be larger emitters of radiation than larger black holes and should shrink and dissipate faster.

Stars of radius smaller than its **Schwarzschild radius**

$$r_s = \frac{2GM}{c^2}$$

$$Mc^2 = \frac{\text{Planck Force}}{2} \times r_s$$

further collapse to produce dark or frozen stars (i.e., the mass of a star is concentrated in a small enough spherical region, so that its mass divided by its radius exceeds a particular critical value, the resulting space-time warp is so radical that anything, including light, that gets too close to the star will be unable to escape its **gravitational grip**). And these dark stars are sufficiently massive and compact and possess a strong gravitational field that prevent even light from escaping out its influence: any light emitted from the surface of the star will be dragged back by the star's gravitational attraction before it could get very far. Such stars become black voids in space and were coined in 1969 by the American scientist **John Wheeler** "the black holes" (i.e., black because they cannot emit light and holes because anything getting too close falls into them, never to return).

Classically, the gravitational field of the black holes (which seem to be among the most ordered and organized objects in the whole universe) is so strong that they would prevent any information including light from escaping out of their influence i.e., any information is sent down the throat of a black hole or swallowed by a black hole is forever hidden from the outside universe (this goes by the statement that "**black holes have no hair**"—that is, they have lost all information, all hair, except for these three parameters: its mass, spin and charge), and all one could say of the gravitational monster what the **poet Dante** said of the entrance to Hell: "**All hope abandon, ye who enter here.**" Anything or anyone who falls through the black hole will soon reach the region of infinite density and the end of time. However, only the laws of classical general relativity does not allow anything (not even light) to escape the gravitational grip of the black hole but the inclusion of **quantum mechanics** modifies this conclusion – quantum fields would scatter off a black hole.

Because energy can be created out of nothing, the pair of short lived virtual particles (one with positive energy and the other with **negative energy**) appears close to the event horizon of a black hole. The gravitational might of the black hole inject energy into a pair of virtual particles ... that tears them just far enough apart so that one with negative energy gets sucked into the hole even before it can annihilate its partner ... its forsaken partner with positive energy... gets an energy boost from the gravitational force of the black hole ... escape outward to infinity (an abstract mathematical concept that was precisely formulated in the work of mathematician **Georg Cantor** in the late nineteenth century)... where it appear as a real particle (and to an observer at a distance, it will appear to have been emitted from the black hole). Because **E= mc squared** (i.e., energy is equivalent to mass), a fall of negative energy particle into the black hole therefore reduces its mass with its horizon shrinking in size. As the black hole loses mass, the temperature of the black hole (which depends only on its mass) rises and its rate of emission of particle increases, so it loses mass more and more quickly. We don't know does the emission process continue until the black hole dissipates completely away or does it stop after a finite amount of time leaving black hole remnants.

The attempt to understand the **Hawking radiation** has a profound impact upon the understanding of the black hole thermodynamics, leading to the description of what the black hole entropic energy is.

- **Black hole energy = 2 × Black hole temperature × Black hole entropy**

$$\frac{Mc^2}{2} = T_{BH} \times S_{BH}$$

$$\frac{Mc^2}{2} = \text{Black hole entropic energy}$$

This means that the entropic energy makes up half of the mass energy of the black hole. For a black hole of one solar mass ( $M = 2 \times 10^{30}$  kg), we get an entropic energy of  $9 \times 10^{46}$  joules – much higher than the thermal entropic energy of the **sun**.

Given that power emitted in Hawking radiation is the rate of energy loss of the black hole:

$$P = -c^2 \frac{dM}{dt} = 2 \times \frac{-dE_s}{dt}$$

The more power a black hole radiates per second, the more entropic energy being lost in Hawking radiation. However, the entropic energy of the black hole of one solar mass is about  $9 \times 10^{46}$  joules of which only  $4.502 \times 10^{-29}$  joules per second is lost in Hawking radiation.

As photons approach the event horizon of a black hole (**light's point of no return**), the light particles with sufficient energy avoid being pulled into the black hole by traveling in a nearly tangential direction known as an **exit cone**. A photon on the boundary of this cone doesn't have enough energy to escape the gravity might of the black hole, and instead they are forced to travel in a circular orbit. These circular orbits of radius

$$r = \frac{3GM}{c^2} = \frac{3r_s}{2}$$

$$Mc^2 = \frac{\text{Planck Force}}{3} \times r$$

are unstable, meaning that after bending through a finite angle around the black hole, the photon will either fall in or bend back out.

The energy of a photon moving in a circular orbit is given by:

$$E = \hbar\omega = \frac{\hbar c}{r}$$

$$E = \frac{\hbar c^3}{3GM}$$

$$E = \frac{8\pi}{3} k_B T_{BH}$$

where:  $k_B$  = Boltzmann constant =  $1.38064852 \times 10^{-23} \frac{J}{K}$ .

$$E = 8.3775 k_B T_{BH}$$

The peak wavelength of this radiation is given by:

$$\lambda_{\max} = \frac{hc}{4.9651 k_B T_{BH}}$$

$$E_{\max} = 4.9651 k_B T_{BH}$$

$$\frac{E}{E_{\max}} = 1.687$$

$$E = \mathbf{1.687} E_{\max}$$

$$E = \frac{\hbar c^3}{3GM} = mc^2$$

$$3mM = (\text{Planck mass})^2$$

Because of this condition the photons orbiting the small black hole carry more mass than those orbiting the big black hole.

$$S_{BH} = k_B \ln W$$

- W is the number of Black hole microstates.

$$W = \exp \left( \frac{Mc^2}{2k_B T_{BH}} \right)$$

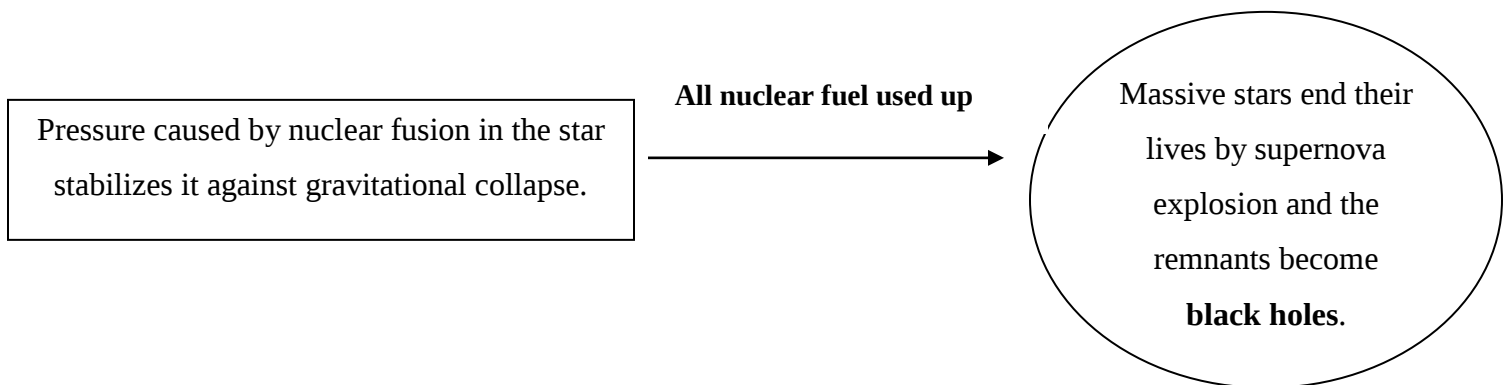
Black hole Entropy:

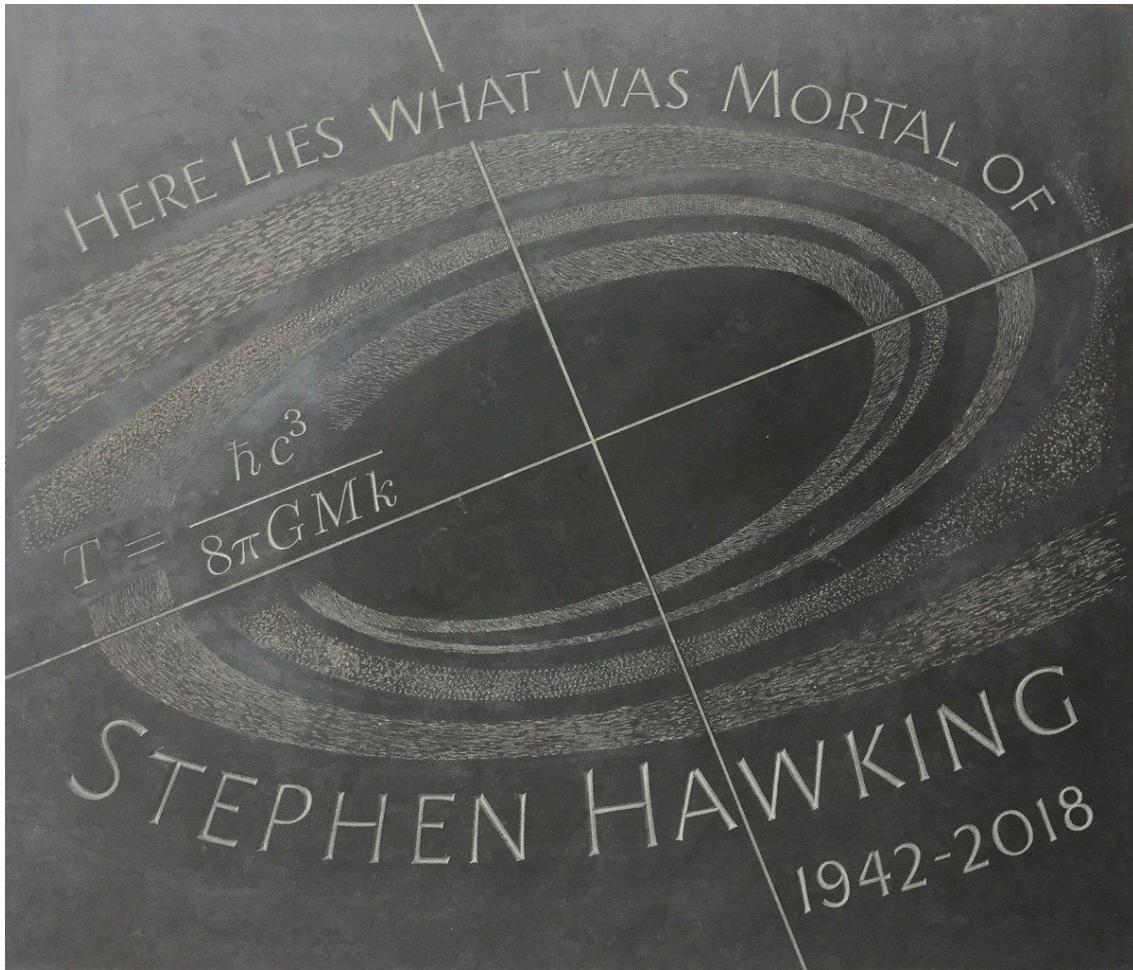
$$S_{BH} = \frac{k_B A}{4 \times \text{Planck area}}$$

where A is the area of the event horizon

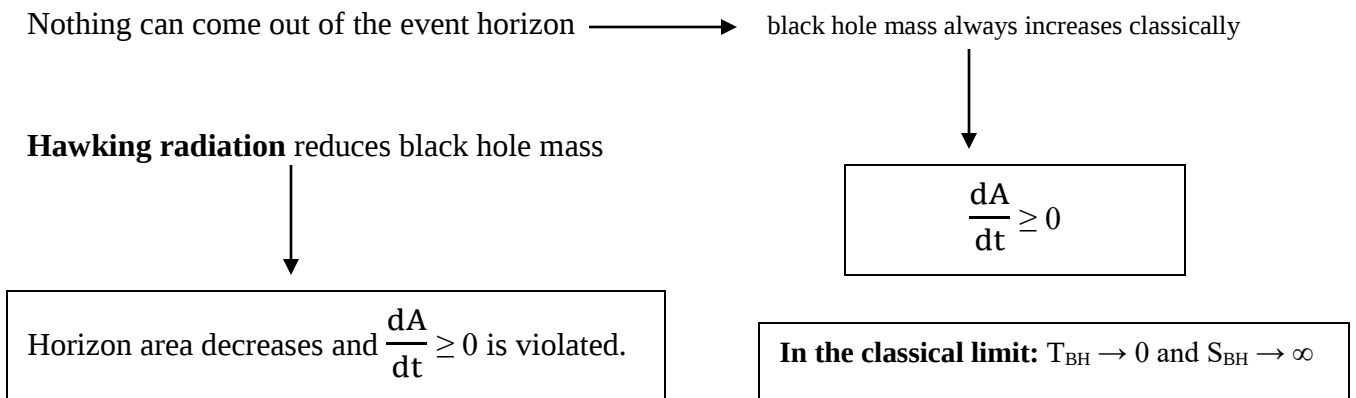
$$W = \exp \left( \frac{A}{4 \times \text{Planck area}} \right)$$

In General theory of relativity, information that goes into a black hole remains inside, and cannot be accessed from outside.





A tombstone of the crypt with Hawking's ashes with the formula ( $T_{\text{BH}} = \frac{\hbar c^3}{8\pi G M k_{\text{B}}}$ ) for the black hole temperature, placed in the flooring at Westminster Abbey in London.



**No hair theorem:**

Black holes are characterized by 3 quantities:

**mass, charge, and angular momentum**

For black hole with 10 solar mass:

Temperature  $\sim 6 \times 10^9$  K (**very low temperature**)

Entropy  $\sim 10^{79} k_B$  (**huge entropy**)

Entropy of sun  $\sim 10^{58}$

**Spin parameter "a"** cannot exceed **black hole mass "M"**: for  $a > M$  the Kerr metric describes a naked singularity.

**Hawking 1975:** Black hole background + Quantum Field theory  $\rightarrow$  Black hole emits radiation!!

**Hawking 1976:** Black hole as a quantum pure state + hawking radiation  $\rightarrow$  Unitarity of Quantum Mechanics is broken!!

- **Solar Mass Black Hole:**  $T_{BH} \sim 10^{-7}$  K,  $t_{\text{evap}} \sim 10^{66}$  yr
- **Black Hole from Large Hadron Collider:**  $T_{BH} \sim 10^{44}$  K,  $t_{\text{evap}} \sim 10^{-80}$  s



|                            |                                            |                                   |
|----------------------------|--------------------------------------------|-----------------------------------|
| Information is indeed lost | Information is stored in Hawking radiation | Evaporation stops at Planck scale |
| Quantum Mechanics is wrong | General Relativity is wrong                | Statistical Mechanics is wrong    |

Quantum Field Theory is valid up to  $10^{-16}\text{cm}$

- Particles collide in three dimensional space.
- As the particles approach in a particle accelerator, their gravitational attraction increases steadily.
- When the particles are extremely close, they may enter space with more dimensions.
- The extra dimensions would allow gravity to increase more rapidly so a black hole can form.
- Such a black hole would immediately evaporate, sending out a unique pattern of thermal radiation.



"This was a golden age, in which we solved most of the major problems in black hole theory even before there was any observational evidence for black holes. In fact, we were so successful with the classical general theory of relativity that I was at a bit of a loose end in 1973 after the publication with George Ellis of our book *The Large Scale Structure of Space–Time*. My work with Penrose had shown that general relativity broke down at singularities, so the obvious next step would be to combine general relativity—the theory of the very large—with quantum theory—the theory of the very small. In particular, I wondered, can one have atoms in which the nucleus is a tiny primordial black hole, formed in the early universe? My investigations revealed a deep and previously unsuspected relationship between gravity and thermodynamics, the science of heat, and resolved a paradox that had been argued over for thirty years without much progress: how could the radiation left over from a shrinking black hole carry all of the information about what made the black hole? I discovered that information is not lost, but it is not returned in a useful way—like burning an encyclopedia but retaining the smoke and ashes.

To answer this, I studied how quantum fields or particles would scatter off a black hole. I was expecting that part of an incident wave would be absorbed, and the remainder scattered. But to my great surprise I found there seemed to be emission from the black hole itself. At first, I thought this must be a mistake in my calculation. But what persuaded me that it was real was that the emission was exactly what was required to identify the area of the horizon with the entropy of a black hole. This entropy, a measure of the disorder of a system, is summed up in this simple formula which expresses the entropy in terms of the area of the horizon, and the three fundamental constants of nature,  $c$ , the speed of light,  $G$ , Newton's constant of gravitation, and  $\hbar$ , Planck's constant. The emission of this thermal radiation from the black hole is now called Hawking radiation and I'm proud to have discovered it."

— Stephen Hawking

- Neutron Star has a hard surface; the curvature is large - but finite.
- Black Hole: No Surface – curvature is infinite at the centre

Black holes are the harmonic oscillator of quantum gravity.

(A. Strominger)

### Stephen Hawking-Jacob Bekenstein debate (1972)

**Stephen Hawking:** the analogy between the area theorem and the second law of thermodynamics is just a matter of coincidence.

**Jacob Bekenstein:** I am not convinced. Nowhere in nature the second law of thermodynamics is violated. Why black holes would be an exception? I believe that the area of black holes is actually a manifestation of their entropy.

**John Wheeler** (Jacob Bekenstein's doctoral advisor) **to Beckenstein:** Your idea is just so crazy that it might actually be true.

**Stephen Hawking:** If a black hole has entropy, it must have a temperature, and if it has a temperature it must radiate like a blackbody. But if nothing can escape from a black hole, how can it radiate?

**1974:** end of the debate and discovery of the Hawking radiation. Professor Stephen Hawking admits that **Jacob Bekenstein** was right: Black Holes have entropy, which is proportional to their area.

According to quantum mechanics, vacuum is no longer "**empty**" as in classical physics. Instead, it is a place where "**virtual particles**" are constantly created and destroyed. The creation of a couple of particle-antiparticle pair of energy  $\Delta E$  is allowed in quantum mechanics as long as their lifetime  $\Delta t$  satisfies the Heisenberg uncertainty principle that can be written as:

$$\Delta E \times \Delta t \geq \frac{h}{4\pi}$$

The larger the energy of the virtual particles, the smaller their survival time is. This quantum-mechanical process of particle-antiparticles creation and destruction is termed as "**vacuum fluctuations**".

A virtual-particle pair has a wave function that predicts that both particles will have opposite spins. But if one particle falls into the black hole, it is impossible to predict with certainty the spin of the remaining particle.

– **Stephen W. Hawking**

The time that the black hole takes to dissipate is:

$$t_{ev} = \frac{5120\pi G^2 M^3}{\hbar c^4} = \frac{480c^2 V}{\hbar G}$$

The power emitted by a black hole in the form of hawking radiation:

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

$$P = \frac{Mc^2}{3t_{ev}} = \frac{2T_{BH} \times S_{BH}}{3t_{ev}}$$

Since:

$$t_{ev} = \frac{480c^2 V}{\hbar G}$$

Therefore:

$$P = \frac{Mc^2}{3t_{ev}} = \frac{G\hbar\rho}{1440}$$

- $\rho = \frac{3M}{4\pi r_s^3} = \frac{3c^6}{32\pi G^3 M^2}$  is the black hole density.

**Heat Capacity:**

$$C = \frac{dQ}{dT}$$

Substituting  $dQ = dM c^2$  and  $T = \frac{\hbar c^3}{8\pi k_B G M}$ :

$$C = - \frac{8\pi k_B G M^2}{\hbar c}$$

$$\text{Specific heat capacity} = - \frac{8\pi k_B G M}{\hbar c}$$

$$\text{Specific heat capacity} = - \frac{c^2}{T_{BH}}$$

Since:

$$\frac{Mc^2}{2} = T_{BH} \times S_{BH}$$

Therefore:

$$\text{Specific heat capacity} = - \frac{2S_{BH}}{M}$$

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2} = \frac{\hbar c}{8\pi G M^2} \times \frac{c^5}{1920 G}$$

Since:

$$C = - \frac{8\pi k_B G M^2}{\hbar c}$$

Therefore:

$$P = - \frac{k_B \times \text{Planck power}}{1920 C}$$

**Black hole entropy:**

$$S_{BH} = \frac{4\pi k_B G M^2}{\hbar c}$$

$$C = -2S_{\text{BH}}$$

Differentiating the equation:  $S_{\text{BH}} = \frac{4\pi k_{\text{B}} GM^2}{\hbar c}$ , we get:

$$dS_{\text{BH}} = \frac{8\pi k_{\text{B}} GM}{\hbar c^3} \times dMc^2$$

$$dMc^2 = T_{\text{BH}} \times dS_{\text{BH}}$$

$$\frac{Mc^2}{2} = T_{\text{BH}} \times S_{\text{BH}}$$

Differentiating the above equation we get:

$$dMc^2 = 2 (T_{\text{BH}} \times dS_{\text{BH}} + dT_{\text{BH}} \times S_{\text{BH}})$$

Since:

$$dMc^2 = T_{\text{BH}} \times dS_{\text{BH}}$$

Therefore:

$$dMc^2 = 2 (dT_{\text{BH}} \times S_{\text{BH}}) + 2dT_{\text{BH}} \times S_{\text{BH}}$$

$$- dMc^2 = 2 (S_{\text{BH}} \times dT_{\text{BH}})$$

- $T_{\text{BH}} = \frac{dMc^2}{dS_{\text{BH}}}$

- $S_{BH} = \frac{1}{2} \times \frac{-dMc^2}{dT_{BH}}$

### Penrose process:

If particle A enters the ergosphere of a Kerr black hole, then the particle A splits into particles B and C.

$$E_A = E_B + E_C$$

$$E_B > E_A \text{ and } E_C < 0$$

The particle B exits the ergosphere with more energy than particle A while particle C goes into the black hole. This process reduces the mass and rotational energy of Kerr black holes and is therefore theorized to cause black hole evaporation.

### Radiation energy density:

$$u = \frac{4}{c} \times \sigma T_{BH}^4 = \frac{4}{c} \times \frac{P}{A}$$

Since:

$$P = \frac{G\hbar p}{1440}$$

Therefore:

$$u = \frac{G\hbar p}{360cA}$$



$$\text{Radiation pressure} = \frac{u}{3} = \frac{G\hbar\rho}{1080cA}$$

"This "Hawking temperature" of a black hole and its "Hawking radiation" (as they came to be called) were truly radical—perhaps the most radical theoretical physics discovery in the second half of the twentieth century. They opened our eyes to profound connections between general relativity (black holes), thermodynamics (the physics of heat) and quantum physics (the creation of particles where before there were none). For example, they led Stephen to prove that a black hole has entropy, which means that somewhere inside or around the black hole there is enormous randomness. He deduced that the amount of entropy (the logarithm of the hole's amount of randomness) is proportional to the hole's surface area. His formula for the entropy is engraved on Stephen's memorial stone at Gonville and Caius College in Cambridge, where he worked.

For the past forty-five years, Stephen and hundreds of other physicists have struggled to understand the precise nature of a black hole's randomness. It is a question that keeps on generating new insights about the marriage of quantum theory with general relativity—that is, about the ill-understood laws of quantum gravity."

— Stephen Hawking

**Conclusion:**

Though the emission of particles from the primordial black holes is currently the most commonly accepted theory within scientific community, there is some disputation associated with it. There are some issues incompatible with quantum mechanics that it finally results in information being lost, which makes physicists discomfort and this raises a serious problem that strikes at the heart of our understanding of science. However, most physicists admit that black holes must radiate like hot bodies if our ideas about general relativity and quantum mechanics are correct. Thus even though they have not yet managed to find a primordial black hole emitting particles after over two decades of searching. Despite its strong theoretical foundation, the existence of this phenomenon is still in question.

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